

AN INTEGRATED APPROACH TO GRAVITY ANOMALY SEPARATION USING SPECTRAL DECOMPOSITION AND PIECEWISE LINEAR ANALYSIS

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ABSTRAK

This paper examines the utilization of gravity methods in the discovery of subsurface geology, focusing on a specific case study conducted in Langsa City. We propose a new technique that combines Spectral Decomposition and Piecewise Linear Fit to separate and analyze regional and residual gravity anomalies. Subsurface geological variations are analyzed using satellite gravity data, and the data is processed using algorithms created on the open-source Python platform. The Piecewise Linear Fit approach is specifically developed to detect change points in gravity data, enabling the differentiation of intricate gravity anomalies. This methodology enhances the precision of distinguishing regional and residual anomalies, leading to a more distinct representation of mass distribution beneath the surface. The utilization of spectral analysis in conjunction with piecewise linear segmentation enables the detection of geological characteristics that are challenging to uncover using traditional techniques. The results indicate that the suggested method is superior in dealing with the difficulties commonly faced in gravity analysis, namely in detecting intricate underground formations such as faults, magmatic intrusions, and geological layer borders. As a result, this method makes important advancements in the field of gravity-based geophysical exploration techniques, providing researchers with a novel and precise instrument to enhance the interpretation of gravity data. This study is anticipated to provide novel guidelines for wider applications in geological mapping and exploration of natural resources.

Keywords: *gravity methods, spectral decomposition, piecewise linear fit, gravity anomalies separation*

1. INTRODUCTION

Gravity anomaly data have wide applications in various geoscience fields, such as natural resource exploration, including groundwater, mineral deposits, geothermal, and petroleum (Cumming, 2016). This method is highly effective for determining subsurface structures by measuring gravity variations caused by differences in the density of subsurface materials (Bechtel, et al., 2014). Gravity anomaly analysis provides deep insights into subsurface mass distribution that cannot be achieved by other geophysical methods, making it a crucial tool in geological mapping and exploration. Gravity anomaly data consists of two main components: regional anomalies and local anomalies. Regional anomalies represent broad gravity variations associated with large geological structures, such as sedimentary basins, mountain ranges, or major tectonic features. These anomalies reflect the effects of density variations on a larger scale and can indicate the presence of significant geological structures. In contrast, local anomalies represent smaller gravity variations related to more specific subsurface structures, such as mineral pockets or oil and gas reservoirs. Local anomalies are more sensitive to small-scale density variations and are more relevant in the context of exploration and development of specific resources.

In the processing of gravity data, it is essential to separate the regional and local anomaly components to clarify more specific subsurface structures. One commonly used technique for this separation is spectral analysis. This technique involves transforming gravity data from the spatial domain to the frequency domain, allowing for the identification and separation of signals originating from sources at different depths. By analyzing the frequency spectrum of gravity data, anomalies caused by density variations at different depths can be identified and separated (Sarkowi & Wibowo, 2021).

The steps of spectral analysis in gravity data processing include several key stages. First, raw gravity data are collected and processed to remove topographic effects and other necessary corrections. Next, the corrected data are transformed into the frequency domain using the Fourier transformation (Caratori Tontini et al., 2007). The resulting frequency spectrum is then analyzed to identify frequency bands associated with regional and local anomalies. Lower frequency bands are typically associated with regional anomalies, while higher frequency bands tend to be related to local anomalies. After this identification, the data can be filtered to separate the regional and local components, allowing further analysis of each anomaly. In spectral analysis, Fourier transformation is used to convert gravity data from the spatial to the frequency domain, resulting in spectral parameters such as wave number (k) in units of cycles per meter versus the logarithm of amplitude ($\ln A$). By plotting this data, a linear relationship is obtained that indicates the

depth of different anomalies. Linear regression of this data is used to identify the depths of anomaly sources, where the slope of the regression line represents variations in the depth of subsurface structures.

The separation between regional and local anomalies in the spectral analysis is often determined by a breakpoint or cut-off point, which represents the boundary between the effects of local and regional anomalies (Gupta & Ramani, 1982). However, the determination of this breakpoint tends to be subjective, depending on the interpretation of individual researchers, which can introduce uncertainty and inconsistency in gravity data analysis. Therefore, a more objective and precise method is needed to reduce this subjectivity and provide more consistent results. The use of the piecewise linear fit technique offers a solution to this challenge by allowing researchers to more accurately determine the boundary between regional and local anomalies. This method divides the frequency spectrum into different linear segments, each representing anomalies at specific depths, and allows for a more objective determination of the breakpoint. Thus, this technique provides a stronger justification and more reliable results following existing theories.

The application of the piecewise linear fit method to gravity data can be performed using code developed with open-source resources such as Python (Chen and Wang, 2013). Python is widely used in various geoscience applications due to its flexibility and wide availability. Examples of applications in the geosciences include geophysical inversion modeling, seismic analysis, geochemical data interpretation, and gravity anomaly separation for resource exploration. By applying this technique, it is expected to provide new insights into the development of gravity applications in various geoscientific fields. This approach not only enhances accuracy in gravity data interpretation but also strengthens the theoretical basis for separating and understanding subsurface structures.

2. METHOD

Gravity data reduction

The corrections involve addressing instrument drift, free-air correction, Bouguer correction, latitude correction, and terrain correction. These corrections involve two contrasting effects: the decrease in gravitational force due to the greater distance between the observation point and the Earth's center (free-air correction), and the increase caused by the gravitational pull of the dense material between this point and sea level (Bouguer correction). The Bouguer anomaly (BA) at a given location is calculated using the equation established by Yamamoto (1999) and is measured in milli Gals (mGal).

$$BA = g - \gamma + \beta h - 2\pi G \rho h + \rho T$$

where g (mGal) is the observed absolute gravity, γ (mGal) is the normal gravity value, βh ($\frac{mGal}{m}$) is the free-air correction, T is the terrain correction per unit average density. G ($m^3 \cdot kg^{-1} s^{-2}$) universal gravitational density, h (m) is elevation terms and ρ ($kg \cdot m^{-3}$) is average density. Following the application of Bouguer and field adjustments using the average density 2.67 g/cm^3 . Terrain modifications were performed with a digital elevation map with a mesh size of 50 meters. After making these adjustments, a complete Bouguer anomaly (CBA) was calculated.

Spectral Analysis

To distinguish between anomalies associated with shallow and deep structures, the CBA was partitioned using a moving average analysis to extract residual anomalies linked to shallow structures and regional anomalies associated with deep structures (Telford et al., 1990). The depth limit of the regional and residual Bouguer anomalies was determined by applying spectrum analysis across numerous processes. Thus, the width of the windows utilized for the Bouguer anomaly filter could be determined. The power spectrum of the Bouguer anomaly is obtained by applying a Fourier transformation to the gravity data, converting it into the frequency domain (Blakely, 1996).

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

where $k = \frac{2\pi}{\lambda}$, dengan k is a wavenumber. The equation can write as follows:

$$F(k) = |F(k)| e^{i\theta k}$$

$$|F(k)| = [Re F(k)^2 + Im F(k)^2]^{\frac{1}{2}}$$

$$\theta = \text{Arctan} \frac{Im F(k)}{Re F(k)}$$

The function $|F(k)|$ represents the amplitude and $\theta(k)$ is a phase of the spectrum. The Fourier transform of the gravitational potential of a point mass can be represented as follows.

$$F(g) = 2\pi G M \frac{e^{|k|(z_0 - z')}}{|k|}$$

When observing the gravitational potential on a horizontal flat plane, the corresponding Fourier transform equation can be expressed as follows:

$$A = Ce^{k(z_0 - z')}$$

Where A is an amplitude and C is a constant. By applying logarithms to the results of the Fourier transform, a correlation can be revealed between the amplitude of the spectrum (A) and the wavenumber (k), as well as the difference in anomalous depth ($z_0 - z'$). Therefore, this relationship can be described by the following equation:

$$\ln A = (z_0 - z')|k| + \ln C$$

Using the equation, we can delineate the boundary of an anomalous source field by plotting a logarithmic graph of the wavenumber (k) against the amplitude ($\ln A$). The slope of this graph directly indicates the depth of the boundary plane of the anomalous source.

Piecewise Linear Function

A piecewise linear function is a type of mathematical function that is defined on an interval of real numbers, divided into several subintervals, on each of which the function is linear or affine. In mathematical terms, an affine function is of the form $f(x) = ax + b$, where a and b are constants. Therefore, a piecewise linear function can be described as a function that is linear on each subinterval of its domain but may have different slopes or intercepts on different subintervals (Chen and Wang, 2013).

A piecewise linear function $f: R \rightarrow R$ can be formally defined as follows:

- The domain of f is divided into a finite or countable collection of intervals, $\{I_i\}$, where each interval $I_i = [x_i, x_{i+1}]$ for $i=0, 1, 2, \dots, n$, or (x_i, x_{i+1}) , or mixed open and closed intervals, depending on the function's domain.
- On each interval, I_i , the function $f(x)$ is defined by a linear expression $f_i(x) = a_i x + b_i$, where a_i and b_i are real numbers specific to that interval. The function is continuous at the boundary points of these intervals if defined as such.
- A piecewise linear function may be continuous or discontinuous. If the endpoints of the subintervals are included and matched properly, the function is continuous across its domain. However, it can also have discontinuities at the boundaries of the subintervals if the function values are not matched at these points.

3. RESULT AND ANALYSIS

The study area is located in Langsa City, Aceh Province, with geographical coordinates ranging from a longitude of 97.840 to 98.093 and a latitude of 4.613 to 4.370. This region is characterized by a variety of geological features that can significantly affect gravity measurements, making it a suitable location for detailed gravity analysis. Gravity data for this study were obtained from satellite measurements accessed through the Topex website (https://topex.ucsd.edu/cgi-bin/get_data.cgi). The data set includes free-air gravity anomalies over the specified coordinates. The free-air anomaly data were then processed to obtain a complete Bouguer anomaly, which provides a clearer picture of subsurface density variations. The initial step in processing the gravity data involved applying several corrections to the raw free-air anomaly data, including (1) Bouguer correction, (2) simple Bouguer correction, and (3) terrain correction. The corrections account for the gravitational effect of the mass between the measurement point and a reference level, assuming a constant average density of 2.67 g/cm³, which is typical for crustal rocks. Then we calculated the Bouguer correction that assumes a flat slab of infinite extent. To refine the Bouguer anomaly, terrain corrections were applied to account for the variations in topography that affect the gravitational field. This step is crucial, especially in areas with significant topographic relief, as it adjusts the Bouguer anomaly to reflect a more accurate gravitational field after removing the effects of local terrain.

After applying these corrections, the complete Bouguer anomaly was calculated, providing a refined data set that is better suited for geological interpretation. The resulting complete Bouguer anomaly data for the study area in Langsa City show a range of gravity values from -37.8 mGal to 5.5 mGal. These values represent variations in the subsurface density that could indicate different geological structures and compositions: The northeastern part of the study area exhibits high gravity anomaly values, which gradually decrease towards the southwest. These high values typically indicate the presence of denser materials in the subsurface, such as igneous or metamorphic rocks, or potentially significant mineral deposits. In deltaic environments, high gravity values could also suggest uplifted basement rocks or the presence of denser alluvial deposits due to sediment compaction or lithification. Conversely, lower gravity anomaly values are found in the southwestern area. These low values could suggest the presence of less dense materials, such as sediments in the deltaic regions, or subsurface voids like those created by karst processes or sediment-filled faults. In a deltaic setting, lower gravity anomalies are often associated with thick sediment accumulations, unconsolidated alluvium, or the presence of fluids like groundwater or hydrocarbons that reduce the overall density.

Spectral analysis was used to better analyze the gravity data and differentiate between regional and local gravity anomalies. This technique entails converting the spatial gravity data into the frequency domain by employing a Fourier transform. By analyzing the frequency composition of gravity anomalies, it is possible to differentiate between the broad, uniform anomalies caused by deep-seated, regional geological features (long-wavelength components) and the

sharper, concentrated anomalies associated with near-surface structures (short-wavelength components). The spectral analysis conducted for the Langsa City area yielded the following findings: The long-wavelength components are indicative of regional anomalies that are probably linked to deeper geological formations, such as uplifted basement or extensive sedimentary basins. Within the context of Langsa, these regional anomalies may indicate the broader structural framework of the area, encompassing significant fault lines or extensive sedimentary sequences. The short-wavelength components correspond to regional anomalies, which may suggest smaller-scale geological characteristics like concentrated mineral deposits, minor faults, or fluctuations in sediment thickness. These anomalies play a crucial role in understanding complicated features of the underlying geology, such as identifying specific areas of mineralization or outlining groundwater reserves.

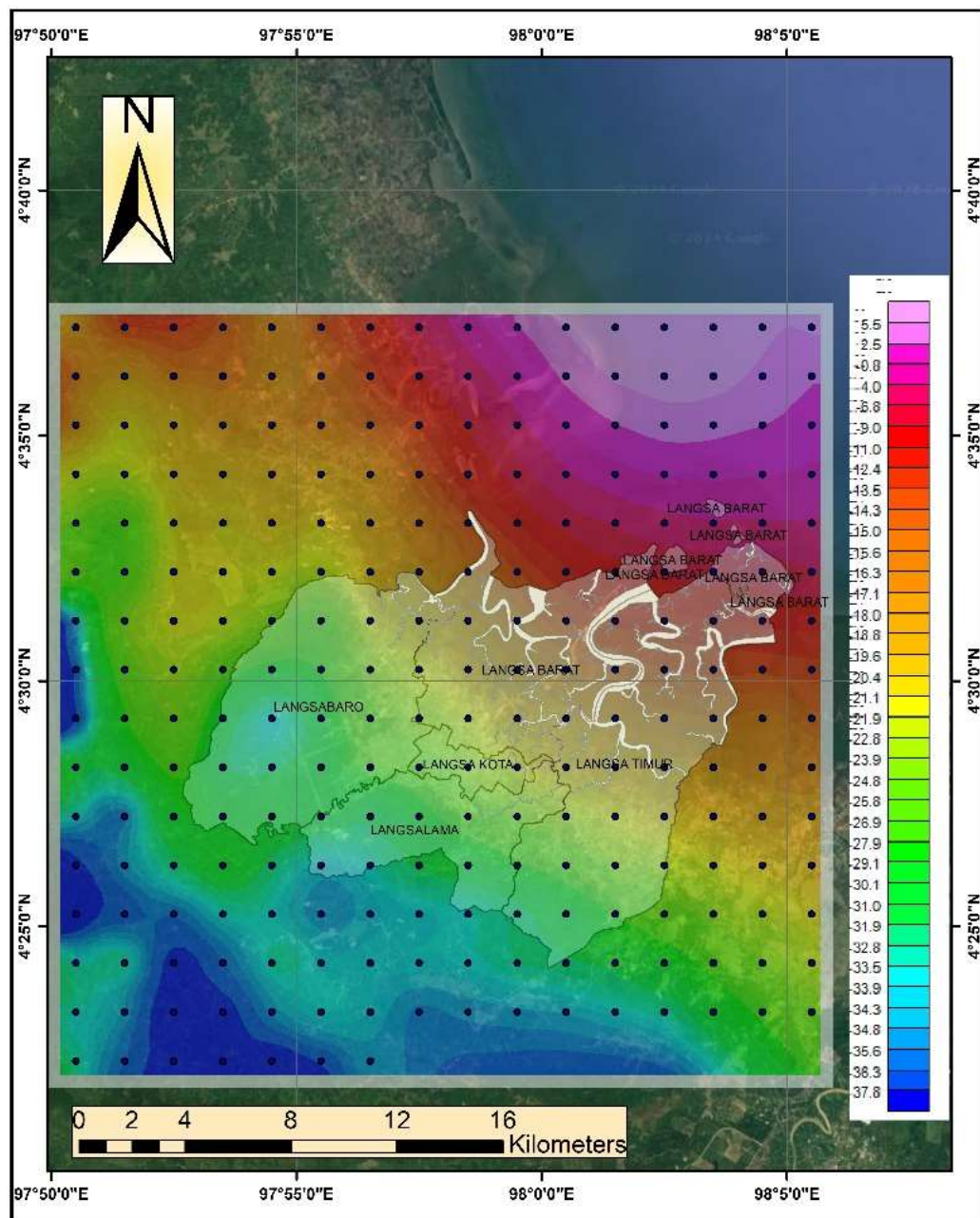


Figure 1. Administrative map of Langsa City with an overlay of the processed complete Bouguer anomaly data and observation station points.

The gravity data analysis, combined with spectral analysis, provides valuable insights into the subsurface geology of Langsa City. The presence of both high and low gravity anomalies suggests a complex geological environment with variations in subsurface density that could be related to a mix of rock types, sediment thickness, and structural features. In deltaic regions like those near Langsa City, high gravity values could suggest areas of denser, compacted sediments or uplifted basement rocks. These zones may also indicate areas where older, denser materials are

closer to the surface, potentially influencing local hydrogeology or mineral exploration prospects. Conversely, low gravity anomalies could point to areas of thicker, less compacted sediment deposits, which may be of interest for groundwater exploration or as indicators of hydrocarbon potential. These low-density zones might also reflect recent sedimentation processes or the presence of softer, less dense materials like clays or organic-rich deposits.

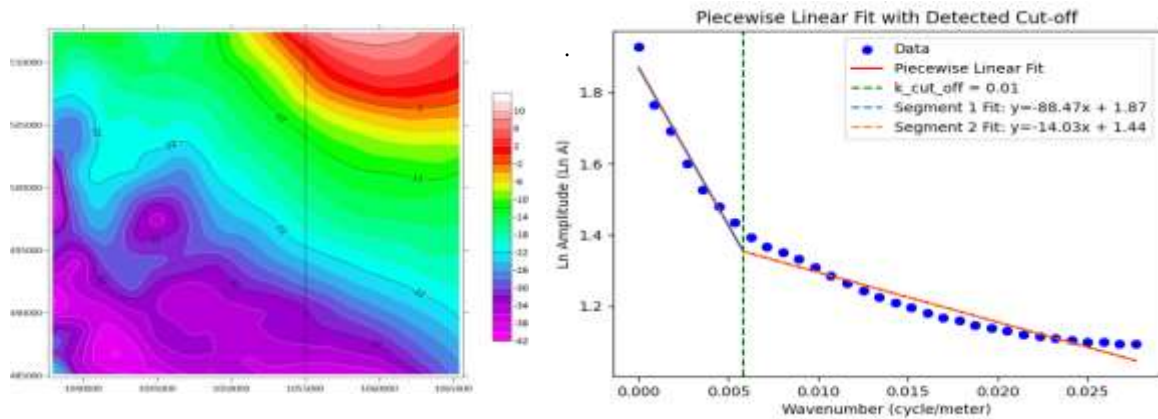


Figure 2. Cross-section of spectral analysis and the spectral curve analyzed using the piecewise linear fit technique

Based on the piecewise linear fit applied to the gravity data from Langsa City, the auto-pick processing resulted in a cut-off wave number (k) value of 0.01, indicated by the green dashed line on the spectral plot. This technique automatically distinguishes between two distinct trends within the data, each representing different types of gravity anomalies. The regional anomaly has been identified with a depth of approximately 88.47 meters, suggesting that this trend is associated with deeper geological structures or features that extend across a broad area. These could be related to large-scale variations in rock density, such as differences in the crustal composition or extensive geological formations. In contrast, the local anomaly is characterized by a shallower depth of about 14.03 meters. This indicates that the local anomalies are related to near-surface features, possibly smaller, discrete geological bodies, or localized variations in sediment thickness or density. By utilizing spectral analysis in conjunction with the piecewise linear fit, we were able to separate the data into regional and residual (local) anomalies. This separation allows for a more nuanced understanding of the subsurface structures, distinguishing between broad, deep-seated geological trends and more localized, near-surface features. The application of the piecewise linear fit in the spectral analysis provides a robust and objective method for initially justifying the separation between regional and local anomalies. This technique improves the clarity of gravity data interpretation, leading to more reliable conclusions about the subsurface geology.

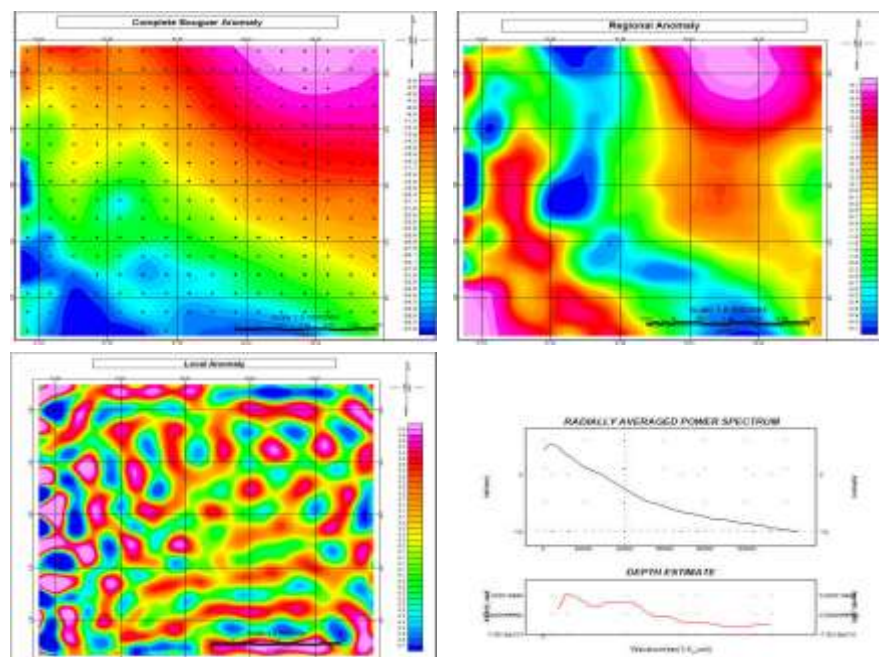


Figure 3. Map of complete Bouguer anomaly, along with regional anomalies, residual/local anomalies, and 2D spectral analysis.

4. CONCLUSION

The use of piecewise linear fit in the spectral analysis of gravity data from Langsa City has proven to be an effective method for differentiating between regional and local gravity anomalies. The auto-pick approach facilitated the objective identification of a cut-off wave number, resulting in the separation of the data into two distinct trends corresponding to different geological features. The calculated depths of the regional and local anomalies provide insights into the subsurface structure, with the regional anomalies representing deeper and more extensive geological formations, while the local anomalies are indicative of near-surface features. This methodological approach enhances the accuracy and objectivity of gravity data interpretation, making it a valuable tool for geophysical studies and resource exploration.

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6. REFERENCES

- Bechtel, T. D., Bosch, F. P., & Gurk, M. (2014). Geophysical methods. In *Methods in karst hydrogeology* (pp. 171-199). CRC Press.
- Blakely, R. J. (1996). Potential Theory in Gravity and Magnetic. In *Cambridge University Press*.
- Caratori Tontini, F., Graziano, F., Cocchi, L., Carmisciano, C., & Stefanelli, P. (2007). Determining the optimal Bouguer density for a gravity data set: implications for the isostatic setting of the Mediterranean Sea. *Geophysical Journal International*, 169(2), 380–388. <https://doi.org/10.1111/j.1365-246X.2007.03340>.
- Chen, D. Z., & Wang, H. (2013). Approximating points by a piecewise linear function. *Algorithmica*, 66(3), 682-713.
- Cumming, W. (2016). Geophysics and resource conceptual models in geothermal exploration and development. In *Geothermal Power Generation* (pp. 33–75). Elsevier. <https://doi.org/10.1016/B978-0-08-100337-4.00003-6>
- Gupta, V. K., & Ramani, N. (1982). Optimum second vertical derivatives in geologic mapping and mineral exploration. *GEOPHYSICS*, 47(12), 1706–1715. <https://doi.org/10.1190/1.1441320>
- Sarkowi, M., & Wibowo, R. C. (2021). Geothermal Reservoir Identification based on Gravity Data Analysis in Rajabasa Area- Lampung. *RISET Geologi Dan Pertambangan*, 31(No. 2), 77–79
- Telford, W. M., Geldart, L. . ., & Sheriff, R. E. (1990). *Applied Geophysics* (Second Edi). Cambridge University Press. https://books.google.co.id/books/about/Applied_Geophysics.html?id=Q8ogAwAAQBAJ&redir_esc=y
- Yamamoto, A. (1999). Estimating the Optimum Reduction Density for Gravity Anomaly: A Theoretical Overview. *Jour. Fac. Sci., Hokkaido Univ., Ser. VII (Geophysics)*, 11(3), 577–599.